Solution

Part 1

Theory:

Consider a small mass m of the liquid at the surface (Figure 4). At dynamic equilibrium

$$N \cos\theta = mg$$

and

$$N \sin\theta = mw^2 x$$

Therefore:

$$\tan\theta = \frac{w^2 x}{g}.$$

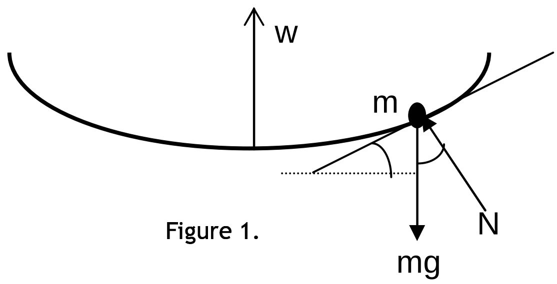

Figure 1.

The profile of the liquid surface can be found as follows:

$$\tan\theta = \frac{dy}{dx}, \quad \frac{dy}{dx} = \frac{w^2 x}{g}$$

so that

$$y = \frac{w^2 x^2}{2g} + y_0$$

where y_0 is the height at $x = 0$.

At a certain point $x = x_0$, height of the liquid h_0 would be the same as if it not rotating. In this case,

$$h_0 = y_0 + \frac{w^2 x_0^2}{2g} \qquad (1)$$

and,

$$x_0^2 = \frac{2g(h_0 - y_0)}{w^2}.$$

Since the volume of the liquid is constant,

$$\pi R^2 h_0 = \int y(2\pi x dx) = 2\pi \int (y_0 + \frac{w^2 x^2}{2g}) x dx ,$$

$$y_0 = h_0 - \frac{w^2 R^2}{4g} \qquad (2)$$

From Eq.1 and Eq.2 one obtains

$$x_0 = \frac{R}{\sqrt{2}}.$$

Experiment:

2R(mm)	*x_0(mm)*	*h_0(mm)*	*H(mm)*
145	51	30	160

$H-h_0$=130 mm

Measure 10T at small speeds and measure 15T-20T at high speeds.

Use $\tan(2\theta)=\dfrac{x}{H-h_0}$ and $w=\dfrac{2\pi}{T}$.

2R(mm)	x_0(mm)
145	51

h_0(mm)	H(mm)	H-h_0(mm)
30	160	130

x(mm)	10T(s)	w(rad/s)	tan(2θ)	θ(rad)	θ(deg)	tan(θ)	w^2(rad/s)^2	
11	21.34	2.94	0.08	0.04	2.4	0.04	8.67	
20	15.80	3.98	0.15	0.08	4.4	0.08	15.81	
26	14.22	4.42	0.20	0.10	5.7	0.10	19.52	
30	12.99	4.84	0.23	0.11	6.5	0.11	23.40	
40	11.74	5.35	0.31	0.15	8.6	0.15	28.64	
51	10.45	6.01	0.39	0.19	10.7	0.19	36.15	
56	9.90	6.35	0.43	0.20	11.7	0.21	40.28	
65	9.40	6.68	0.50	0.23	13.3	0.24	44.68	
70	9.08	6.92	0.54	0.25	14.2	0.25	47.88	
85	8.39	7.49	0.65	0.29	16.6	0.30	56.08	
100	7.71	8.15	0.77	0.33	18.8	0.34	66.41	
112	7.43	8.46	0.86	0.36	20.4	0.37	71.51	
132	7.00	8.98	1.02	0.40	22.7	0.42	80.57	
61.4	11.19	6.20	0.47	0.21	11.98	0.21	41.51	Ave.

The last line is for error calculation only.

The slope of the Figure 5 is 0.0052 (s/rad)^2 which gives

$$g=\frac{x_0}{slope}=\frac{5.1}{0.0052}=980\,\mathrm{cm/s}^2 .$$

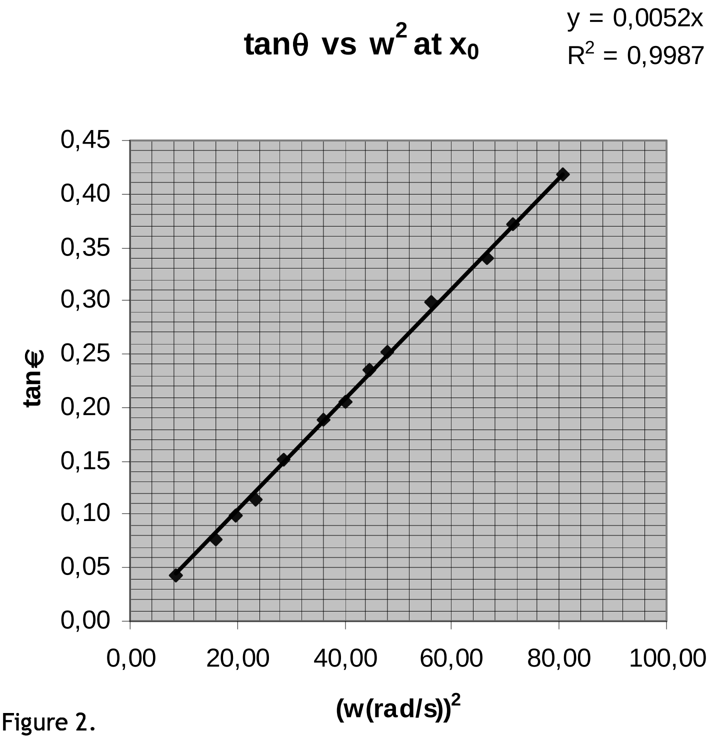

Figure 2.

Error Calculation (possible methods):

$$g = \frac{w^2 x_0}{\tan\theta}$$

$$\frac{\Delta g}{g} = \sqrt{4\left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta x_0}{x_0}\right)^2 + \left(\frac{\Delta(\tan\theta)}{\tan\theta}\right)^2} \quad \frac{\Delta w}{w} = \frac{\Delta T}{T}$$

$$\frac{\Delta(\tan\theta)}{\tan\theta} \approx \frac{\Delta\theta}{\theta}$$

(since from the table $\tan\theta \cong \theta$)

$$\theta \approx \frac{x}{H - h_0}, \quad \frac{\Delta\theta}{\theta} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta H + \Delta h_0}{H - h_0}\right)^2}$$

$$\frac{\Delta g}{g} = \sqrt{4\left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta x_0}{x_0}\right)^2 + \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta H + \Delta h_0}{H - h_0}\right)^2}$$

Using the values H=160 mm, ΔH=1mm, h_0=30 mm, Δh_0=1mm, x_{av}=61.4 mm, Δx_{av}=1mm, T_{av}=1.1s, ΔT=0.01 s, x_0=51 mm, Δx_0=1mm one obtains

g = 980±34 cm/s^2

- Note that from the method of least squares one obtains the following results:

  g = 982 cm/s^2 with a standard deviation of **σ= 33 cm/s^2**

- From the linear regression of the data slope tanθ vs w^2 is found to be 0.052 with a standard error of 5.14x10^{-5}, therefore:

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta(slope)}{slope}\right)^2 + \left(\frac{\Delta x_0}{x_0}\right)^2} = 0.02$$

g = 980±20 cm/s^2

Part 2a

H(mm)	10T(s)	w(rad/s)	lnw	H-h_0(mm)	ln(H-h_0)
158	10.31	6.09	0.784921	128	2.107
209	13.19	4.76	0.677935	179	2.253
190	11.70	5.37	0.729994	160	2.204
150	9.80	6.41	0.806954	120	2.079
129	9.21	6.82	0.83392	99	1.996
119	8.75	7.18	0.856172	89	1.949
110	8.10	7.76	0.889695	80	1.903

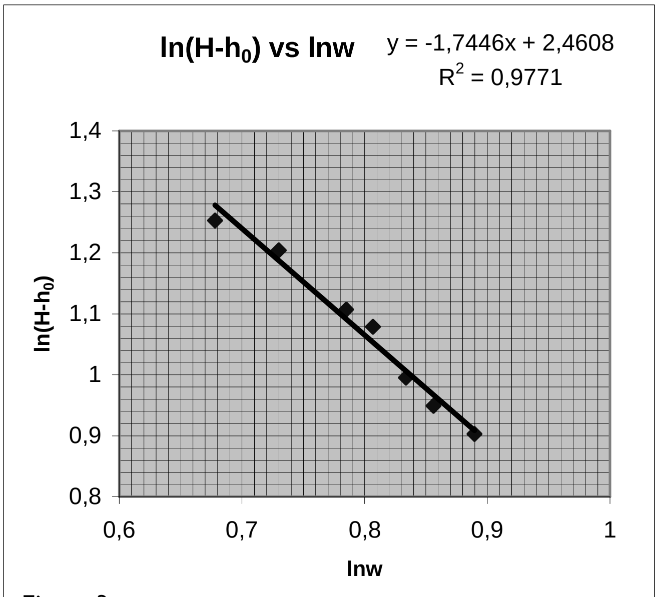

Figure 3.

Thus the focal length depends on w as

$$f = Aw^{n},$$

and

$$n \sim -1.7.$$

The plot of H-h_0 vs. 1/w^2 is also acceptable as a correct plot.

Part 2b

ω Range(rad/s)	Orientation	Variation of the size	Image
ω=0	ER		V
0<ω<8.2* 0<ω<6.3**	ER	D	V
8.2<ω<14.6* 6.3<ω<14.0**	INV	I	R
14.6<ω<ω_{max}* 14.0<ω<ω_{max}**	ER	NC	V

* for H=110 mm
** for H=240 mm

ω values depend on the initial values of H, h_0, etc.

Note that measurements only at one H value are required from the students.

Part 3

Measurement of wavelength

Both the grating and the screen are in air. Normal incidence.

Screen to grating distance : L
Distance between the diffraction spots seen on the screen : x
Order of diffraction : m

- L= 225 mm, x_{av}=77 mm for m=±1 d=1/500 mm

  $$\tan\alpha = \frac{x_{av}}{L} = \frac{77}{225}$$

  $$\lambda = \frac{1}{500}\sin\alpha = 647\ \text{nm}$$

- L= 128 mm, x_{av}=44 mm for m=±1, d=1/500 mm

  $$\tan\alpha = \frac{44}{128}$$

  $$\lambda = \frac{1}{500}\sin\alpha = 650\ \text{nm}$$

- L= 128 mm, x_{av}=111 mm for m=±2, d=1/500 mm

  $$\tan\alpha = \frac{111}{128}$$

  $$\lambda = \frac{1}{2x500}\sin\alpha = 655\ \text{nm}$$

The average value of λ is λ_{av}=651 nm.

Measurement of refractive index

2R=145 mm

Distance between the spots measured on the curved screen = $R\alpha$

$R\alpha_{av}$ = 17 mm for m=±1 α_{av} = 0.234 rad

using $n = \dfrac{m\lambda}{d\sin(\alpha)}$, one obtains n=1.40

If the curvature of the screen is neglected:

$$\tan\alpha = \frac{17}{72.5}$$

$$\alpha = 13.20^\circ$$

$$n = \frac{\lambda}{d\sin(\alpha)} = \frac{651(nm)}{\frac{1}{500}(mm)x10^6\sin(\alpha)} = 1.43$$

Grading Scheme for Experimental Competition

Part 1	**7.5 pts**
• Derivation of Equation 1	1.0 pts
• Calculation of ω using period measurements	1.0 pts
At low speeds 10T is OK	
At high speeds 20T is expected	***-0.2 pts***
Missing units	***-0.2 pts***
• Calculation of $\tan 2\theta$, $\tan\theta$ at each ω	1.0 pts
Calculation of $\tan 2\theta$	0.5 pts
Calculation of $\tan\theta$	0.5 pts
• Plot of $\tan\theta$ vs ω^2	1.5 pts
Axes with labels and units	0.4 pts
Drawing best fit line	0.5 pts
At least 6 different data in a wide range of ω	0.6 pts
No. of measurements 5:	***-0.2 pts***
No of measurements 4:	***-0.4 pts***
No of measurements 3 or less:	***-0.6 pts***
• Calculations	2.0 pts
calculation of slope with unit	1.0 pts
calculation of g	1.0 pts
FULL credit for	
$9.3<g<10.3\ m/s^2$ ($\pm$5% error)	
For g values credits to be subtracted from the total credit of 7.5:	
$10.3<g<10.5 m/s^2$, $9.1<g<9.3 m/s^2$	***-0.5 pts***
$8.8<g<9.1\ m/s^2$, $10.3<g<10.8\ m/s^2$	***-1.0 pts***
outside the above ranges	***-1.5 pts***
• Error Calculation	1.0 pts

Part 2a

5.5 pts

• Measurements of H vs ω	<u>0.6 pts</u>
Calculation of ω using period measurements	0.4 pts
At low speeds 10T is OK	
At high speeds 20T is expected	***-0.2 pts***
H-ω table	0.2 pts
• Plot of F vs ω	<u>2.4 pts</u>
Calculation of $F=H-h_0$	0.5 pts
Plot with axis labels	0.8 pts
Drawing best fit line	0.5 pts
At least 6 different data in a wide range of ω	0.6 pts
No. of measurements 5:	***-0.2 pts***
No of measurements 4:	***-0.4 pts***
No of measurements 3 or less:	***-0.6 pts***
• Calculations	<u>2.5 pts</u>
Calculation of slope with unit	1.0 pts
Dependence $F\alpha\ 1/\omega^2$	1.5 pts

Part 2b

3.5 pts

• Every correct item in the table	0.25 pts

Part 3

3.5 pts

(At least 3 measurements at different orders are required)

• Wavelength measurement	<u>1.2 pts</u>
Distance measurements and calculation of angle	0.6 pts
Calculation of λ	0.6 pts
Credits to be subtracted from the total credit of 1.2:	
If λ is outside the range 600-700 nm	***-0.4 pts***
If less then 3 measurements	***-0.4 pts***
• Measurement of n	<u>2.3 pts</u>
Distance measurements and calculation of angle	0.6 pts
Realizing λ/n	0.8 pts
Calculation of n	0.9 pts
credits to be subtracted from the total credit of 2.3:	
If n is outside range 1.3-1.6	***-0.4 pts***
If less then 3 measurements	***-0.4 pts***