Barem de evaluare şi de notare

Orice altă rezolvare care conduce la rezultate corecte se va puncta corespunzător

Nr. item	*Problema a III-a Proprietăți ale materialelor paramagnetice*		Punctaj
1.	Pentru:		**1,0p**
	intensitatea curentului electric echivalent $i = \frac{e}{T} = \frac{v}{2\pi \cdot r} \cdot e$	0,2p	
	modulul momentului magnetic $\lvert\vec{\mu}\rvert = \pi \cdot r^2 \cdot \frac{v}{2\pi \cdot r} \cdot e = \frac{e}{2m^*}\lvert\vec{L}\rvert$	0,5p	
	$\vec{\mu} = -\frac{e}{2m^*}\vec{L}$	0,2p	
2.	Pentru:		**1,0p**
	$\cos\theta_m = \frac{m}{\sqrt{l \cdot (l+1)}}$	0,5p	
	$\theta_m = \arccos\left[\frac{m}{\sqrt{l \cdot (l+1)}}\right]$, cu $m \in Z$, $-l \le m \le l$	0,5p	
3.	Pentru:		**1,0p**
	expresia energiei medii $\langle E\rangle = \frac{\sum_{k=1}^{i} E_k \cdot \exp(-E_k/k_BT)}{\sum_{k=1}^{i} \exp(-E_k/k_BT)}$	0,5p	
	$\begin{cases} \langle E\rangle = \dfrac{\delta_1 \cdot k_B \cdot T\left(1 \cdot e^{-\delta_1} + 2 \cdot e^{-2\delta_1} + 3 \cdot e^{-3\delta_1}\right)}{e^{-\delta_1} + e^{-2\delta_1} + e^{-3\delta_1}} \\ \langle E\rangle = \dfrac{\delta_1 \cdot k_B \cdot T\left(1 + 2 \cdot e^{-\delta_1} + 3 \cdot e^{-2\delta_1}\right)}{1 + e^{-\delta_1} + e^{-2\delta_1}} \end{cases}$	0,5p	
4a.	Pentru: proiecția momentului cinetic pe direcția câmpului magnetic aplicat		**0,5p**
	$\lvert\vec{\mu}\rvert_z = \left(e/\left(2m^*\right)\right) \cdot g \cdot \lvert\vec{J}\rvert_z$	0,2p	
	$\mu_z = \left(e/\left(2m^*\right)\right) \cdot g \cdot m \cdot \hbar = \mu_B \cdot g \cdot m$, $-j \le m \le j$	0,3p	
4b.	Pentru: energia asociată atomului aflat în câmp magnetic		**0,5p**
	$E_m = -\vec{\mu} \cdot \vec{H} = \left(e/\left(2m^*\right)\right) \cdot g \cdot \vec{J} \cdot \vec{H} = g \cdot \mu_B \cdot H \cdot m$, $-j \le m \le j$	0,5p	
4c.	Pentru: valoarea medie a proiecției momentului magnetic		**2,0p**
	$\langle\mu\rangle = \frac{\sum_{m=-j}^{j} \mu_B \cdot g \cdot m \cdot \exp(-E_m/k_BT)}{\sum_{m=-j}^{j} \exp(-E_m/k_BT)} = \frac{\sum_{m=-j}^{j} \mu_B \cdot g \cdot m \cdot \exp(-g \cdot \mu_B \cdot H \cdot m/k_BT)}{\sum_{m=-j}^{j} \exp(-g \cdot \mu_B \cdot H \cdot m/k_BT)}$	0,2p	

	$\langle\mu\rangle = \frac{k_BT}{H}\cdot\frac{\sum_{m=-j}^{j} x\cdot m\cdot exp(-x\cdot m)}{\sum_{m=-j}^{j} exp(-x\cdot m)} = \frac{k_BT}{H}\cdot\frac{\vartheta}{Z}$, unde $x = \frac{g\cdot\mu_B\cdot H}{k_BT}$ 0,2p $Z = e^{x\cdot j} + e^{x\cdot(j-1)} + \ldots + 1 + \ldots + e^{-x\cdot(j-1)} + e^{-x\cdot j} = e^{x\cdot j}\cdot\frac{1-e^{-x\cdot(2j+1)}}{1-e^{-x}}$ 0,2p $Z = \frac{e^{x\cdot\left(j+\frac{1}{2}\right)} - e^{-x\cdot\left(j+\frac{1}{2}\right)}}{e^{\frac{x}{2}} - e^{-\frac{x}{2}}} = \frac{sinh\left[x\cdot\left(j+\frac{1}{2}\right)\right]}{sinh\left(\frac{x}{2}\right)}$ 0,2p $\vartheta = x\cdot j\cdot e^{x\cdot j} + x\cdot(j-1)\cdot e^{x\cdot(j-1)} + \ldots + 0 - \ldots - x\cdot(j-1)\cdot e^{-x\cdot(j-1)} - x\cdot je^{-x\cdot j}$ 0,2p $\vartheta = x\cdot\frac{dZ}{dx}$ 0,2p $\vartheta = x\cdot\frac{\left(j+\frac{1}{2}\right)\cdot cosh\left[x\cdot\left(j+\frac{1}{2}\right)\right]\cdot sinh\left(\frac{x}{2}\right) - \frac{1}{2}\cdot cosh\left(\frac{x}{2}\right)\cdot sinh\left[x\cdot\left(j+\frac{1}{2}\right)\right]}{sinh^2\left(\frac{x}{2}\right)}$ 0,2p $\langle\mu\rangle = \frac{k_BT\cdot x}{H}\cdot\frac{\left(j+\frac{1}{2}\right)\cdot ctgh\left[x\cdot\left(j+\frac{1}{2}\right)\right]\cdot sinh\left(\frac{x}{2}\right) - \frac{1}{2}\cdot cosh\left(\frac{x}{2}\right)}{sinh\left(\frac{x}{2}\right)}$ 0,2p $\langle\mu\rangle = g\cdot\mu_B\cdot\left\{ctgh(x) - \frac{1}{2}\cdot ctgh\left(\frac{x}{2}\right)\right\}$, pentru $j = 1/2$ 0,2p $\langle\mu\rangle = g\cdot\mu_B\cdot\left\{\frac{3}{2}\cdot ctgh\left(\frac{3}{2}\cdot x\right) - \frac{1}{2}\cdot ctgh\left(\frac{x}{2}\right)\right\}$, pentru $j = 1$ 0,2p	
4d.	Pentru: expresia magnetizării $M = N\cdot\langle\mu\rangle = N\cdot g\cdot\mu_B\cdot\left\{\left(j+\frac{1}{2}\right)\cdot ctgh\left[x\cdot\left(j+\frac{1}{2}\right)\right] - \frac{1}{2}\cdot ctgh\left(\frac{x}{2}\right)\right\}$ 0,3p câmpuri magnetice mari, la saturație, când $x\to\infty$, $ctgh(x) = 1$ $M = N\cdot\langle\mu\rangle \cong N\cdot g\cdot\mu_B\cdot j$ 0,6p	**0,9p**
4e.	Pentru: câmpuri magnetice foarte mici, pentru care $x\to 0$ $M(x\to 0) = N\cdot\frac{g^2\cdot\mu_B^2\cdot H}{3k_BT}\cdot\left(j^2+j\right)$ 0,6p	**0,6p**

4f.	Pentru: expresia energiei medii pe particulă $\langle E\rangle = \dfrac{\sum_{k=1}^{n} E_k \cdot exp(-E_k/k_BT)}{\sum_{k=1}^{n} exp(-E_k/k_BT)} = \dfrac{\sum_{k=1}^{n} E_k \cdot exp(-yE_k)}{\sum_{k=1}^{n} exp(-yE_k)}$, unde $y=\dfrac{1}{k_BT}$ 0,2p $\eta = \dfrac{d(\langle E\rangle)}{dT} = \dfrac{d(\langle E\rangle)}{dy}\cdot\dfrac{dy}{dT} = -\dfrac{1}{k_BT^2}\cdot\dfrac{d(\langle E\rangle)}{dy}$ 0,3p $\dfrac{d(\langle E\rangle)}{dy} = \dfrac{d}{dy}\left(\dfrac{\sum_{k=1}^{n} E_k \cdot exp(-yE_k)}{\sum_{k=1}^{n} exp(-yE_k)}\right)$ 0,2p $\dfrac{d(\langle E\rangle)}{dy} = \dfrac{\left[-\sum_{k=1}^{n} E_k^2 \cdot exp(-yE_k)\right]\cdot\left[\sum_{k=1}^{n} exp(-yE_k)\right]+\left[\sum_{k=1}^{n} E_k \cdot exp(-yE_k)\right]^2\cdot\left[\sum_{k=1}^{n} E_k \cdot exp(-yE_k)\right]}{\left[\sum_{k=1}^{n} exp(-yE_k)\right]^2}$ 0,3p $\dfrac{d(\langle E\rangle)}{dy} = -\langle E^2\rangle + \langle E\rangle^2$ 0,3p $\eta = \dfrac{1}{k_BT^2}\cdot\left(\langle E^2\rangle - \langle E\rangle^2\right)$ 0,2p	**1,5p**
4g.	Pentru: $E_m << k_B \cdot T$, toate exponențialele $exp(-E_m/k_BT) \cong exp(0)$ sunt practic egale cu unitatea 0,2p $\langle E\rangle = \dfrac{\sum_{m=-j}^{j} E_m}{2j+1} = g\cdot\mu_B\cdot H\cdot\dfrac{\sum_{m=-j}^{j} m}{2j+1} = 0$ 0,3p $\begin{cases}\langle E^2\rangle = \dfrac{\sum_{m=-j}^{j} E_m^2}{2j+1} = (g\cdot\mu_B\cdot H)^2\cdot\dfrac{\sum_{m=-j}^{j} m^2}{2j+1} = (g\cdot\mu_B\cdot H)^2\cdot\dfrac{2\sum_{m=1}^{j} m^2}{2j+1} \\ \langle E^2\rangle = (g\cdot\mu_B\cdot H)^2\cdot\dfrac{j\cdot(j+1)\cdot(2j+1)}{3(2j+1)} = (g\cdot\mu_B\cdot H)^2\cdot\dfrac{j\cdot(j+1)}{3}\end{cases}$ 0,3p expresia căldurii specifice pe particulă, pentru materialul paramagnetic $\eta = \dfrac{j\cdot(j+1)}{3}\cdot\dfrac{(g\cdot\mu_B)^2}{k_B}\cdot\dfrac{H^2}{T^2}$ 0,2p	**1,0p**
TOTAL Problema a III-a		**10p**

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